Fibre Tracking, MLS

$$\min_F \int_{\mathbb{R}^3} K(y-x)\left(T(y)-F(y-x)\right)^2 dy \qquad \circledast$$

Rewrite in transformed coordinates:

$\bar{y} = R_x^{-1}(y-x)$

where R_x denotes the matrix (e_1, e_2, e_3) of eigenvectors of $T(x)$

$$\circledast \iff \min_F \int_{\mathbb{R}^3} \underbrace{K(R_x^{-1}\bar{y})}_{=K(\bar{y},\lambda)} \Big(T(x+R_x^{-1}\bar{y}) - \underbrace{F(R_x^{-1}\bar{y})}_{=\sum_{i,j,k=0}^{N} F_{ijk}\, \bar{y}_1^i \bar{y}_2^j \bar{y}_3^k} \Big)^2 d\bar{y}$$

$\lambda = (\lambda_1, \lambda_2, \lambda_3)^T$ eigenvalues of $T(x)$

expressed by polynomials

$\bar{y} = (\bar{y}_1, \bar{y}_2, \bar{y}_3)^T$

Tensor of order 2

also different basis can be chosen

$\iff$ differentiate with respect to $\frac{\partial}{\partial F_{ijk}^{mn}}$ and set to zero

Solve $\sum_{u,v,w=0}^{N} M_{uvw,ijk} F_{uvw}^{mn} = G_{ijk}^{mn}$, $\quad m,n = 1,2,3$

where $M_{uvw,ijk} = \int_{\mathbb{R}^3} \bar{y}_1^{i+u} \bar{y}_2^{j+v} \bar{y}_3^{w+k} K(\bar{y},\lambda)\, d\bar{y}$

$$G_{ijk}^{mn} = \int_{\mathbb{R}^3} T^{mn}(x+R_x^{-1}\bar{y})\, \bar{y}_1^i \bar{y}_2^j \bar{y}_3^k\, K(\bar{y},\lambda)\, d\bar{y}$$

$\Rightarrow \bar{T}(x) = F(0) = F_{000}$

A sensible choice for K is

$$K(\bar{y},\lambda) = \gamma\, e^{-\left(\frac{\bar{y}_1^2}{\sigma^2\lambda_1} + \frac{\bar{y}_2^2}{\sigma^2\lambda_2} + \frac{\bar{y}_3^2}{\sigma^2\lambda_3}\right)}$$

where σ^2 is a constant variance and γ a normalization constant

Streamline Integration (Discretization):

$$x(t_{k+1}) = x(t_k) + \bar{e}_1(x(t_k))\,\Delta t \quad , \; \Delta t = t_{k+1} - t_k$$

$\bar{e}_1(x(t_k))$ is the eigenvector to $\lambda_1(x(t_k))$ of $\bar{T}(x(t_k))$. $\bar{T}(x(t_k))$ is the smoothed tensor obtained by MLS using the kernel $K(\bar{y}, \lambda)$ for eigenvalus $\lambda = (\bar{\lambda}_1(x(t_{k-1})), \bar{\lambda}_2(x(t_{k-1})), \bar{\lambda}_3(x(t_{k-1})))^T$ and corresponding eigenvector $\bar{e}_1, \bar{e}_2, \bar{e}_3$ of $\bar{T}(x(t_{k-1}))$.