Übungen zu Numerische Methoden II

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Exercise Sheet 1

1. Let X be the binomial random variable with $n = 20$. Given the null-hypothesis $H_0 : p = 0.5$ and the alternative hypothesis $H_1 : p = 0.3$, calculate the first and the second order error, if the rejection (critical) region is given by

$$G = \{x : x \leq 2\}.$$

2. Consider the test whether or not a coin is balanced based on the number of heads Y on 36 flips of the coin. Define $H_0 : p = c$ and $H_1 : p \neq c$. If the rejection region is given by $G = \{x : |x - 18| \geq 4\}$, what is:

   (a) the first order error, if $c = 0.5$.

   (b) the second order error, if $c = 0.7$.

   Hint: In both exercises, consider the MATLAB *code from Exercise 6 (Ex. sheet 6) to compute the sums.*

3. Consider a random sample of n observations with density function

$$f(x \,|\, \theta) = \frac{1}{2\theta^3} x^2 e^{-x/\theta}, \quad \text{for} \quad x > 0, \quad \theta > 0.$$

   Suppose the hypothesis $H_0 : \theta = \theta_0$ is tested against $H_1 : \theta = \theta_1$, assuming $\theta_1 > \theta_0$. Compute the constant k such that the rejection region is written as

$$G = \{x : \bar{x} > k\}.$$

4. Consider the random sample $Y_1, ..., Y_N$ having a Bernoulli-distribution with parameter p,

$$f(y_i \,|\, p) = p^{y_i}(1 - p)^{1-y_i}, \quad y_i = 0, 1.$$

Suppose that the hypothesis $H_0 : p = p_0$ is tested against $H_1 : p = p_1$, where $p_1 > p_0$. Then, the rejection region is given by

$$G = \{y_i : \frac{L(p_0)}{L(p_1)} < k\}.$$

Show that G exists if and only if

$$\sum_{i=1}^{n} y_i > k^*, \quad \text{for some constant} \quad k^* := k^*(k).$$

5. Assume the following model

$$y_i = \beta_0 + \beta_1 x_{i1} + ... + \beta_k x_{ik} + \epsilon_i, \quad i = 1, ..., n$$

to represent n independent observations y_i related to the independent variables x_{ik}. In matrix form, it is equivalently written as

$$\vec{y} = X\vec{\beta} + \vec{\epsilon},$$

where

$$X = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nk} \end{pmatrix}.$$

Given the data,

x	-1	0	1	2	3
y	3	2	1	1	0.5

compute the least-squares estimator $\vec{\beta}$ if:

(a) $\vec{y} = \beta_0 + \beta_1 x + \epsilon$ (linear model).

(b) $\vec{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon$ (quadratic model).

6. Consider the 31 data points $(x_1, y_1), ..., (x_{31}, y_{31})$ as given in EX6.TXT. Implement in MATLAB a program for plotting a line and a parabola fitting the data, using the linear and the quadratic model (exercise 5), equivalently.