Exercise Sheet 7

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Exercise 30. Let $K : L^2([0,1]) \to L^2([0,1])$ be the integral operator generated by the kernel

$$k(s,t) := \begin{cases} s(1-t) & \text{if } s \leq t, \\ t(1-s) & \text{if } s > t. \end{cases}$$

Find the eigenvalues of K. (<u>Tip</u>: Calculate $(Kx)''$.)

Exercise 31. Show that the integral operator K of Exercise 30 is positive definite.

Exercise 32. Show that there is a kernel $k(G \times G)$ with $k(s,s) \geq 0$ such that the corresponding integral operator is self adjoint but not positive semi-definite. (Thus the converse direction of Lemma 2.46 (ii) does not hold.)