Photoacoustic and Coupled Physics Imaging

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(joint work with Peter Elbau, Andreas Kirsch, Rainer Schulze)

There exist various reconstruction formulas and back-projection algorithms for *photoacoustic imaging* (see the survey [4] and the references therein). Also different measurement devices for the ultrasound pressure have been suggested. Most common are small detectors based on materials, which exhibit a strong piezoelectric effect and can be immersed safely in water (i.e. polymers such as PVDF). In analytical reconstruction formulas, they are considered point detectors. Other experimental setups have been realized with line and area detectors, which collect averaged pressure (see [9] for a survey).

Here, we consider the problem of photoacoustic *sectional imaging.* Opposed to standard photoacoustic imaging, where the detectors record sets of two-dimensional projection images from which the three-dimensional imaging data can be reconstructed, *single slice imaging* reconstructs a set of two-dimensional slices, each by a single scan procedure. The advantages of the latter approach are a considerable increase in measurement speed and the possibility to do selective plane imaging. In general, this can only be obtained by the cost of decreased out-of-plane resolution. The difference in this model to previously studied models is that the wave propagation is considered fully three-dimensional, the initialization *and* measurements are fully two-dimensional due to the selective plane illumination and detection. Therefore, such setups require novel reconstruction formulas, which have been explained in this talk. After the introduction of the *universal* back-projection algorithm introduced in [11] this goal seems superfluous. However, it has been shown recently by Natterer [6] that *universal* back-projection is only exact for special sampling geometries. Here, for sliced imaging and certain sampling setups, we can indeed find mathematically exact universal reconstruction algorithms for arbitrary strictly convex sampling domains Ω.

We model the sectional photoacoustic imaging by assuming that the *initial pressure distribution* $f : \mathbb{R}^3 \to \mathbb{R}$ is perfectly focused in the illumination plane $\{x \in \mathbb{R}^3 \mid x_3 = 0\}$:

$$f(\xi, z) = \hat{f}(\xi)\delta(z), \quad \xi \in \mathbb{R}^2,\ z \in \mathbb{R}, \tag{1}$$

for some smooth function $\hat{f} : \mathbb{R}^2 \to \mathbb{R}$. The resulting pressure wave $p : \mathbb{R}^3 \times [0, \infty) \to \mathbb{R}$, we assume to be the solution of the linear three-dimensional wave equation

$$\boxed{\begin{aligned} \partial_{tt} p(\xi, z; t) &= \Delta_{\xi,z} p(\xi, z; t), \\ \partial_t p(\xi, z; 0) &= 0, \\ p(\xi, z; 0) &= f(\xi, z) = \hat{f}(\xi)\delta(z) \end{aligned}} \tag{2}$$

for all $\xi \in \mathbb{R}^2$, $z \in \mathbb{R}$, and $t > 0$. Here,

$$\Delta_{\xi,z} = \partial_{\xi_1\xi_1} + \partial_{\xi_2\xi_2} + \partial_{zz}$$

denotes the three-dimensional Laplacian in Euclidean coordinates.

The goal of sectional imaging is to recover the function $\hat{f}$, describing the initial pressure distribution, from certain measurements of the pressure wave p. The position of the detectors performing these measurements shall be given by the boundary $\partial\Omega$ of a convex domain $\Omega \subset \mathbb{R}^2$ in the illumination plane, where we additionally assume that $\hat{f}$ has compact support in Ω.

Sectional imaging has become experimentally tractable [5, 10, 2, 3]. The following mathematical work also shows that it can be used to determine other imaging modalities, such as the wave speed variations by photoacoustic measurements, by using backprojection formulas. Opposed to *sectional 3D photoacoustic imaging*, where stacks of complementary two-dimensional projection images are produced, the proposed approach for simultaneous imaging consists in performing overlapping sliced imaging by rotation and translation of the specimen. This also generates enough data for reconstructing the two independent parameter functions, speed of sound and absorption density. Exact reconstruction formulas for the wave speed function in ultrasound reflectivity tomography are based on the *Born approximation* to the wave equation, which is valid for moderately varying speed of sound. Reconstruction formulas for ultrasound reflectivity tomography have already been derived by Norton & Linzer [7, 8] and are also based on inversion formulas for the spherical mean operator. The possibility of exact inversion in both fields supports to derive exact inversion formulas for both parameters, which is outlined in this talk.

The reconstruction formulas for simultaneous imaging utilize techniques from reflectivity imaging and photoacoustic imaging. In the current state of research we can provide exact reconstruction formulas, but use way too much data. The practical criticism with our approach is that, due to physical constraints, sectional imaging is applicable to elongated objects. Current sectional photoacoustic imaging experiments, which, however, do not perform wave speed estimation, sample along a cylinder and do not require to steer the sections in all angular directions. In this case, however, currently, there do not exist analytical back-projection formulas for both parameters, the absorption density and wave speed function. Numerical reconstruction methods based on regularization, similar as proposed in [13, 12], have to be implemented. For practical applications we are thinking of an intermediate model, where a cylinder containing the specimen is slightly tilted. This experimental suggestion does not contradict with the assumptions of sectional imaging. In fact these experiments could also be used in combination with a photoacoustic spiral tomograph approach (see [1], which however did not elaborate on simultaneous identification of the wave speed and absorption density, but only on the later). This paper serves as a case study for deriving more complex reconstruction formulas for advanced sampling geometries. We intend to report on numerical studies in a follow up paper. Finally we mention that P. Elbau (RICAM, Linz, Austria) has recently discovered that one measurement in time for each section is sufficient to reconstruct both parameters - in his approach, although using significantly less data, the complex steering of the experiment is still required, only the time measurements can be performed shorter.

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