A1

$$J(x,\gamma) = \int_0^L C(s)\,\mu(s)\, e^{-\int_0^s \mu(t)\,dt}\, ds$$

$$C(s) = R(s) = k_d\, C_\ell\, C_0(s)\big(N(s)\cdot L(s)\big) + k_s\, C_\ell\, \big(N(s)\cdot H(s)\big)^p + k_a C_a$$

general

a) $$C(s) = R(s) = k_d\, C_{\ell_1}\, C_0(s)\big(N(s)\cdot L_1(s)\big) + k_s\, C_{\ell_1}\big(N(s)\cdot H_1(s)\big)^p$$
$$+ k_d\, C_{\ell_2}\, C_0(s)\big(N(s)\cdot L_2(s)\big) + k_s\, C_{\ell_2}\big(N(s)\cdot H_2(s)\big)^p$$

b) as a) but $C_{\ell_i} = C_{\ell_i}(s) = \tilde{C}_{\ell_i}\, e^{-\int_s^{D_i} \mu(t)\,dt}$

c) $$C(s)\mu(s) = \int_{S^2} W(\gamma(s),\xi)\, J(\gamma(s),\xi)\, dS(\xi)$$

$$J(x,\gamma) = J_0(\gamma(L),\gamma')\, e^{-\int_0^L \mu(t)\,dt} + \int_0^L \int_{S^2} W(\gamma(s),\xi)\, J(\gamma(s),\xi)\, dS(\xi)\, e^{-\int_0^s \mu(t)\,dt}\, ds$$

$\gamma(0) = x$

solves $$\frac{dJ(\ell)}{d\ell} = \underbrace{C(\ell)\mu(\ell)}_{\text{emission, Reflection}} - \underbrace{\mu(\ell)\, J(\ell)}_{\text{absorption}}$$

How can we incorporate a self-emitting particle in a), b)

$$C(s) = R(s) + E(s)$$

direction of $\gamma(s)$

emission direction

$$E(s) = k_e\, C_e(s)\big(\gamma'(s)\cdot D(s)\big)$$

A2

$$\tilde{J}(x_{kl}) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\, f(x_{kl}-x_{ij}) = J_{kl}$$

$$\Rightarrow \underbrace{\begin{pmatrix} f(x_{11}-x_{11}) & \cdots & f(x_{11}-x_{1N}) & \cdots & f(x_{11}-x_{NN}) \\ \vdots & & & & \vdots \\ f(x_{1N}-x_{11}) & & & & f(x_{1N}-x_{NN}) \\ \vdots & & & & \vdots \\ f(x_{NN}-x_{11}) & \cdots & f(x_{NN}-x_{1N}) & \cdots & f(x_{NN}-x_{NN}) \end{pmatrix}}_{=G} \underbrace{\begin{pmatrix} a_{11} \\ \vdots \\ a_{1N} \\ \vdots \\ a_{NN} \end{pmatrix}}_{=a} = \begin{pmatrix} J_{11} \\ \vdots \\ J_{1N} \\ \vdots \\ J_{NN} \end{pmatrix}$$

$$\Rightarrow a = G^{-1} J = \begin{pmatrix} \sum_{k=1}^{N}\sum_{l=1}^{N} g_{11,kl} J_{kl} \\ \vdots \\ \sum_{k=1}^{N}\sum_{l=1}^{N} g_{NN,kl} J_{kl} \end{pmatrix}$$

$$\Rightarrow \tilde{J}(x) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\, f(x-x_{ij})$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} \left(\sum_{k=1}^{N}\sum_{l=1}^{N} g_{ij,kl} J_{kl} \right) f(x-x_{ij})$$

$$= \sum_{k=1}^{N}\sum_{l=1}^{N} \underbrace{\left(\sum_{i=1}^{N}\sum_{j=1}^{N} g_{ij,kl}\, f(x-x_{ij}) \right)}_{K(x,x_{kl})} J_{kl}$$

A31

$$J(t) = J_0 e^{-\int_0^t f(r)\,dr} + \int_0^{t_1} g(s)\, e^{-\int_s^t f(r)\,dr}\, ds + \int_{t_1}^{t_2} g(s)\, e^{-\int_s^t f(r)\,dr}\, ds + \ldots + \int_{t_{n-1}}^{t} g(s)\, e^{-\int_s^t f(r)\,dr}\, dr$$

$$= J_0\, e^{-\int_0^t f(r)\,dr} + e^{-\int_{t_1}^t f(r)\,dr} \int_0^{t_1} g(s)\, e^{-\int_s^{t_1} f(r)\,dr}\, ds$$

$$+ \ldots$$

$$= e^{-\int_{t_1}^t f(r)\,dr} \left(J_0\, e^{-\int_0^{t_1} f(r)\,dr} + \int_0^{t_1} g(s)\, e^{-\int_0^{t_1} f(r)\,dr}\, ds \right)$$

$$+ \ldots$$

$$= e^{-\int_{t_1}^t f(r)\,dr} \left(J_0\, e^{-\int_0^{t_1} f(r)\,dr} + \int_0^{t_1} g(s)\, e^{-\int_0^{t_1} f(r)\,dr}\, ds \right)$$

$$+ e^{-\int_{t_2}^t f(r)\,dr} \int_{t_1}^{t_2} g(s)\, e^{-\int_s^{t_2} f(r)\,dr}\, ds$$

$$+ \ldots$$

$$= e^{-\int_{t_2}^t f(r)\,dr} \Bigg[\overbrace{e^{-\int_{t_1}^{t_2} f(r)\,dr}}^{=T_2} \overbrace{\left(J_0\, \overbrace{e^{-\int_0^{t_1} f(r)\,dr}}^{=T_1} + \int_0^{t_1} g(s)\, e^{-\int_0^{t_1} f(r)\,dr}\, ds \right)}^{=J_1} + \int_{t_1}^{t_2} g(s)\, e^{-\int_s^{t_2} f(r)\,dr}\, ds \Bigg]$$

$$+ \ldots$$