Exercise Sheet 5 (Dezember 18th, 2015).

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Exercise 17.

Find a scaling $t = [t]t^*$, and $x = [x]x^*$ in a way that the convection diffusion equation

$$\frac{\partial u}{\partial t}(t,x) = -d\frac{\partial^2 u}{\partial x^2}(t,x) - v\frac{\partial u}{\partial x}(t,x) \quad \text{for } t > 0,\ x \in \mathbb{R}$$

with positive constants d, and v will have the following form:

$$\frac{\partial u}{\partial t^*}(t^*,x^*) = \frac{\partial^2 u}{\partial x^{*2}}(t^*,x^*) - Pe\frac{\partial u}{\partial x^*}(t^*,x^*).$$

Exercise 18. Solve the following 1D-Poisson equation (a) analytically (b) numerically using Finite Differences (FD). The Poisson equation is given by

$$\frac{\partial^2}{\partial x^2}u(x) = -1 \quad \text{for } x \in [0,1]$$

with boundary conditions

$$u(0) = 0, \quad u(1) = 0.$$

Compare your the exact solution with the numerical solution for $h = 0.1$ and $h = 0.001$.

Exercise 19. The function

$$u^*(x,y) = \sin(xy)$$

solves the 2D-Poisson equation

$$-\Delta u = f$$

for

$$f = sin(xy)(x^2 + y^2),$$

and Dirichlet boundary values from $u^*(x,y)$ the domain $\Omega = [0,1] \times [0,1]$. Solve the discrete system of equation with $h = 1/8, 1/16$, and $1/32$. Compare the global error of the approximated solution.

Exercise 20. Solve the 1D heat equation

$$\frac{\partial}{\partial t}u(x,t) = \frac{\partial^2}{\partial x^2}u(x,t) \quad \text{for } x \in [0,1]$$

with the boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(1,t) = 0 \quad \text{for all } t > 0$$

and initial condition

$$u(x,0) = \begin{cases} 0 & \text{for } x \in [0,1/4) \cup (3/4,1] \\ 1 & \text{for } x \in [1/4,3/4] \end{cases}$$

with the implicit and explicit Euler using the grid size $h = 0.1$ and step size $\tau = 0.001$, and $h = 0.01$, $\tau = 0.01$. Compare the results of the methods at $t = 0.1$.