Exercise Sheet 2 (October 30th, 2015)

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Exercise 5. Use your code from Exercise 4 to solve the ODE

$$\dot{y} = 3y^{2/3}\,, \qquad y(-1) = 1\,,$$

and determine the numerical order of the methods at the final times $T = -0.5, 0, 1$. Explain the results.

Exercise 6. Implement in MATLAB Taylor's method. Apply it to solve numerically the ODE from Exercise 3 with initial data $y(0) = 1$. Test your implementation also for

$$\dot{y} = y(t)\cos(t)\,, \qquad y(0) = 1\;.$$

In the next exercises, we consider a mass-spring system, which can lead to a stiff ODE depending on the relative size of the input parameters. The second order system of ODEs is given by the equations

$$\begin{aligned} y_1'' &= -\frac{k_1}{m_1}y_1 + \frac{k_2}{m_1}(y_2 - y_1)\,, \\ y_2'' &= -\frac{k_2}{m_2}(y_2 - y_1)\,, \end{aligned}$$

with initial conditions

$$\begin{aligned} y_1(0) &= a, \quad y_1'(0) = b\,, \\ y_2(0) &= c, \quad y_2'(0) = d\;. \end{aligned}$$

Here, y_1 and y_2 describe the displacement of the masses m_1 and m_2 from the equilibrium position, and k_1 and k_2 the spring constants.

Exercise 7. Rewrite the second order mass-spring system as a first order system by introducing auxiliary variables for y_1' and y_2'.

Exercise 8. Write a MATLAB implementation of the classical Runge-Kutta method and use it for computing numerical solutions of the mass-spring system with the following parameters:

1. Non-stiff setting: $k_1 = 100, a = 0, k_2 = 200, b = 1, m_1 = 10, c = 0, m_2 = 5, d = 1$.
2. Stiff setting: $k_1 = 100, a = 0, k_2 = 2000, b = 1, m_1 = 10, m_2 = 0.1, c = 0, d = 1$.

Choose the time domain $[0, 10]$, and solve the systems for different numbers of steps and compare the results (for the stiff setting, the comparison of the steps between 500 and 502 can be interesting, that is with step-size 0.02 and 0.0199 respectively.) In addition, test your implementation on the linear ODE

$$y' = -200(y - \sin(t)) + \cos(t)\;. \tag{1}$$