**Exercise 1.** Find the general solution of the differential equation

\[ \dot{y} = y^2. \]

In addition, find particular solutions for the initial conditions \( y(1) = 1, y(1) = -1 \) and \( y(1) = 0 \), respectively.

**Exercise 2.** Find the general solution of the differential equation

\[ (t^2 + 1)\dot{y} + ty = \frac{1}{2}. \]

**Exercise 3.** Find the general solution of the differential equation

\[ (3t - y)\dot{y} + t = 3y. \]

Note that this equation is of homogeneous type.

**Exercise 4.** Implement in MATLAB the explicit Euler method, the midpoint method and Heun’s method for the solution of a system of ODEs of the form

\[ \dot{y}(t) = f(t, y(t)), \quad y(t_0) = y_0, \tag{1} \]

up to some time \( T > t_0 \).

Test your code on the system of ODEs

\[ \dot{y}_1 = -y_2, \quad \dot{y}_2 = y_1, \quad y_1(0) = 1, \quad y_2(0) = 0. \]

with final time \( T = 2\pi \), and compute the approximation error \( \text{err}_j \) (the norm of the difference between the solution of the ODE and its approximation) for step sizes \( 2\pi/2^j, j = 1, ..., 10 \) (the actual solution is: \( y_1(t) = \cos(t), y_2(t) = \sin(t) \)). A numerical approximation of the order of the methods (the numerical order) can be found by observing the ratios

\[ -\frac{\ln(\text{err}_{j+1}/\text{err}_j)}{\ln 2} \text{ for large } j. \]

Explain why this is reasonable and determine the numerical order.