Exercises Inverse Problems, SS 2010

Exercise Sheet 6

Markus Haltmeier

Exercise 27. Let $k \in L^2(G \times G)$ be such that the associated integral operator $K : L^2(G) \to L^2(G)$ is self adjoint, and let $(\lambda_i, x_i)_{i \in \mathbb{N}}$ be an eigensystem of K. Show that the family

$$\big((s,t) \mapsto x_i(s)\bar{x}_j(t)\big)_{(i,j)\in\mathbb{N}^2}$$

is an orthonormal basis of a closed subspace W of $L^2(G \times G)$, and that $k \in W$.

Exercise 28. Show that a compact self adjoint operator $K\colon H \to H$ is positive semi-definite, if and only if all eigenvalues are non-negative.

Exercise 29. Let $k \in C(G \times G)$ and let the associated integral operator $K\colon L^2(G) \to L^2(G)$ be positive semi-definite. Show that $k(s,s) \geq 0$ for all $s \in G$.