**Exercise 1** (Discrete Convolution). A discrete image of size $n_1 \times n_2$ is a family

$$ u := (u(i_1, i_2))_{(i_1, i_2) \in I} \in \mathbb{R}^{n_1 \times n_2}, $$

where $I = \{1, \ldots, n_1\} \times \{1, \ldots, n_2\}$. The discrete convolution $u * v \in \mathbb{R}^{n_1 \times n_2}$ of two images $u \in \mathbb{R}^{n_1 \times n_2}$ and $v \in \mathbb{R}^{m_1 \times m_2}$ is defined by

$$ (u * v)(i_1, i_2) := \sum_{j_1=1}^{m_1} \sum_{j_2=1}^{m_2} u(i_1 - j_1 + \frac{m_1 + 1}{2}, i_2 - j_2 + \frac{m_2 + 1}{2}) v(j_1, j_2). \quad (1) $$

Here $m_1, m_2$ are assumed to be odd numbers, and $\sigma$.

(a) Illustrate the definition of the discrete convolution (for simplicity take $n_2 = m_2 = 1$).

(b) Is the discrete convolution as defined in (1) symmetric, associative, and/or distributive (with respect to addition)?

(c) Write a Matlab function `conv_same(u, v)` that computes the discrete convolution as defined in (1).

(d) Test the function `conv_same(u, v)` with

$$ u = \text{double(imread('lena512.bmp'))} \quad \text{and} \quad v = [-1; 0; 1]. $$

**Exercise 2** (Gauss Filter). A basic method for denoising an image $u$ is to compute the discrete convolution with the Gaussian kernel $k_{\sigma, m} \in \mathbb{R}^{m \times m}$,

$$ k_{\sigma, m}(j_1, j_2) := \frac{1}{\sigma \sqrt{2\pi}} \exp \left( - \frac{(j_1 - (m + 1)/2)^2 + (j_2 - (m + 1)/2)^2}{2\sigma^2} \right), \quad \text{for} \ (j_1, j_2) \in \{1, \ldots, m\}. $$

Write a Matlab function

$$ \text{filt_gauss}(u, \text{sig}, m) $$

that convolves an image $u$ with the Gaussian kernel.

**Exercise 3** (Edge Detection). One widely used method for edge detection is to apply the Sobel operator:

$$ G : u \mapsto \sqrt{(u * G_x)^2 + (u * G_y)^2}, $$

where

$$ G_x := \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}, \quad G_y := \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}. $$

Write a Matlab function

$$ \text{edge_sobel}(u) $$

that applies the Sobel operator to an image $u$. Test `edge_sobel(u)` with

$$ u = \text{double(imread('lena512.bmp'))}. $$