1. Consider the trapezoidal method
\[ y_{i+1} = y_i + \frac{t_{i+1} - t_i}{2} [f(t_i, y_i) + f(t_{i+1}, y_{i+1})], \]
to approximate the solution of the initial value problem
\[ y'(t) = f(t, y(t)), \quad t \in [a, b], \quad y(t_0) = y_0, \]
in \( n + 1 \) equidistant points in \([a, b]\). Solve the initial value problem
\[ y'(t) = y(t) - t^2 + 1, \quad t \in [0, 1], \quad y(0) = \frac{1}{2} \]
for \( n = 2 \) using the implicit (backward) Euler method and the trapezoidal method.

2. Consider the following Runge-Kutta arrays
\[
\begin{array}{c|ccc}
0 & 0 & 0 & 0 \\
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
\hline
\frac{1}{2} & \frac{1}{2} & 0 & 1
\end{array}
\quad \text{and} \quad
\begin{array}{c|ccc}
0 & 0 & 0 & 0 \\
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
\hline
\frac{1}{2} & \frac{1}{2} & 0 & 1
\end{array}
\]
which define two second-order Runge-Kutta methods for approximating the solution of the initial value problem
\[ y'(t) = -y(t), \quad t > 0, \quad y(0) = 1 \]
For a given \( h > 0 \), find for both arrays the coefficients \( C(h) \), such that the corresponding method takes the form
\[ y_{i+1} = C(h) y_i \]

3. Consider the Runge-Kutta method with tableau
\[
\begin{array}{c|ccc|c|ccc}
\frac{1}{2} - \frac{\sqrt{3}}{6} & \frac{1}{4} & \frac{1}{4} - \frac{\sqrt{3}}{6} & \frac{1}{2} \\
\frac{1}{2} + \frac{\sqrt{3}}{6} & \frac{1}{4} + \frac{\sqrt{3}}{6} & \frac{1}{4} & \frac{1}{2}
\end{array}
\]
Show that this method is A-stable.
4. Let the linear system of ODEs
\[
y_1(t) = -100y_1(t), \quad y_1(0) = 1, \\
y_2(t) = -2y_2(t) + y_1(t), \quad y_2(0) = 1.
\]
Characterize the above system with respect to stiffness.

5. Implement in MATLAB the Euler method and the trapezoidal method (ex. 1) to approximate the exact solution \(y(t) = e^{t - t^2/2}\) of the initial value problem
\[
y'(t) = (1 - t)y(t), \quad y(0) = 1, \quad t \in [0, 2],
\]
for \(h := t_{i+1} - t_i = 0.5, 0.2 \text{ and } 0.1\).

6. The fourth-order Runge-Kutta method is given by
\[
y_0 = y(a), \\
k_0 = hf(t_i, y_i), \\
k_1 = hf(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_0), \\
k_2 = hf(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_1), \\
k_3 = hf(t_{i+1}, y_i + k_2), \\
y_{i+1} = y_i + \frac{1}{6}(k_0 + 2k_1 + 2k_2 + k_3), \quad i = 0, ..., n - 1.
\]
Implement in MATLAB the above method to approximate the solution of the initial value problem
\[
y'(t) = -\frac{y(t)}{1 + t}, \quad t \in [0, 1], \quad y(0) = 1, \quad \text{for } h = 0.005.
\]