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Photoacoustic Imaging taking into account Attenuation

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The difficult issue of effects of and corrections for the attenuation of acoustic waves in *photoacoustic imaging* has been studied [8, 1, 10, 6], although no complete conclusion on the feasibility of this models has been reached. Mathematical models of attenuation are formulated in the frequency domain, since the attenuation is known to be strongly frequency dependent. Let G_0, G be the Green functions of the wave equation and the attenuated wave equation, respectively. The common *attenuation model* reads as follows:

$$\mathcal{F}G(x,\omega) = \exp\left(-\beta(|x|,\omega)\right)\mathcal{F}G_0(x,\omega)\ . \tag{1}$$

Here $\mathcal{F}$ denotes the Fourier transform with respect to time, ω is the frequency, and x is the space coordinate. The complex function $\beta(|x|,\omega)$ is called the *attenuation coefficient*. Well known models, are *power laws*, Szabo's model [11, 12], and the thermoviscous wave equation (see e.g. [4]), which are characterized by different functions β.

Distinctive features of unattenuated wave propagation (i.e., the solution of the standard wave equation) are *causality* and *finite front wave speed*. It is reasonable to assume that the attenuated wave satisfies these distinctive properties as well.

The standard photoacoustic imaging problem consists in backpropagation of waves $p(s,t)$, where s is an element of the recording surface, to $f(x) = p(x,0)$, where $x \in \Omega$, and Ω is domain of interest, bounded by the measurement surface. Thereby p is considered the solution of the wave equation

$$\frac{\partial^2 p}{\partial t^2}(x,t) = \Delta p\,, \quad x \in \mathbb{R}^3,\ t > 0$$

with initial conditions

$$p(x,0) = f(x)\,, \quad \frac{\partial p}{\partial t}(x,0) = 0\,, \quad x \in \mathbb{R}^3\ .$$

The parameter f is the imaging parameter in photoacoustics. For a series of methods for backpropagation we refer to [7]. If attenuation is taken into account, and equation (1) is considered the basic model for attenuation, then the imaging problem decouples into the standard photoacoustic imaging problem for the wave equation and a deconvolution problem with kernel $\mathcal{F}^{-1}\left(\exp\left(-\beta(|x|,\cdot)\right)\right)(t)$.

[3] state "Power attenuation laws have been used in phenomenological acoustics because of their extreme simplicity as well as their conformity with the physical requirements of causality and dissipativity." However, as it is also been stated in [3], causality and dissipativity restricts the frequency dependence of attenuation in a power-law medium where $A = Const \times |\omega|^\alpha$ to $0 < \alpha < 1$. This can also be deduced in a mathematically rigorous way from a distribution theory [2]. In contrast to previous work the powerful mathematics of distribution theory allows to prove or disprove causality very efficiently.

Power laws with exponent greater than one are of relevance in photoacoustic imaging, since for biological specimens and oils, the power law index has been experimentally identified to be larger than one.

Inversion techniques based on an un-physical model are questionable. We therefore propose using an approximate power law [6]: The philosophy behind this approach is to calculate an attenuation law, which approximates a power law in the frequency spectrum where it has been experimentally validated and it is extended outside of the measured spectrum in such a way that the wave model becomes causal. Based on the results from [6], we developed such approximation models that satisfy the needs of causality and in addition have a finite front wave speeds.

A work, which is concerned with causal attenuated wave equations, which starts modeling at constitutive laws is [9]. There the derived equations are defined via *relaxation* and currently cannot simulate power laws with fractional index.

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